MATH 231
Exercise 4

1. For the graph $G$ of Figure 1, determine the cut-vertices, bridges, and blocks of $G$.

Figure 1

2. Let $G$ be the Petersen graph. Give an example of
(a) a minimum vertex-cut in $G$ and
(b) a vertex-cut $U$ in $G$ such that $U$ is not a minimum vertex-cut of $G$ and no proper subset of $U$ is a vertex-cut of $G$.

3. Give an example of a graph $G$ with the following properties.
(a) $\kappa(G) = 2$, $\lambda(G) = 3$, and $\delta(G) = 4$.
(b) $\kappa(G) = 2$, $\lambda(G) = 2$, and $\delta(G) = 3$.

4. In the graph $H$ of Figure 2, the vertices represent street intersections.
(a) What is the maximum number $k$ such that if road repairs are done at the same time to any $k$ roads (making use of these roads impossible), then it is still possible to travel between every two intersections?
(b) What is the maximum number $k$ such that if intersection repairs are done at the same time to any $k$ intersections (making use of these intersections impossible), then it is still possible to travel between every two intersections that are not under repair?

Figure 2