MATH231  
Exercise 6

1. There are positions open in seven different divisions of a major company; advertising (a), business (b), computing (c), design (d), experimentation (e), finance (f), and guest relation (g). Six people are applying for some of these positions, namely:
   - Alvin (A): a, c, f;
   - Beverly (B): a, b, c, d, e, g;
   - Connie (C): c, f;
   - Donald (D): b, c, d, e, f, g;
   - Edward (E): a, c, f;
   - Frances (F): a, f.

   (1) Represent this situation by a bipartite graph.

   (2) Is it possible to hire all six applicants for six different positions?

2. Figure 1 shows two bipartite graphs $G_1$ and $G_2$, each with partite sets $U = \{v, w, x, y, z\}$ and $W = \{a, b, c, d, e\}$. In each case, can $U$ be matched to $W$?

   ![Figure 1](image)

   Figure 1

3. Determine the values of $\alpha(H), \beta(H), \alpha_v(H)$ and $\beta_e(H)$ for the graph $H$ of Figure 2. Give an example of a minimum vertex cover, a maximum independent set of vertices, a minimum edge cover, and a maximum independent set of edges of $H$.

   ![Figure 2](image)

   Figure 2
4. Determine which of the cubic graphs $G_1, G_2$ and $G_3$ in Figure 3 (1) has a 1-factor, (2) is 1-factorable.

Figure 3

5. Show that $C_n \times K_2$ is 1-factorable for $n \geq 4$. 